Exercise 24

Differentiate.

$$F(t) = \frac{At}{Bt^2 + Ct^3}$$

Solution

Use the quotient rule to differentiate $F(t)$.

$$\begin{aligned}
F'(t) &= \frac{d}{dt}\left(\frac{At}{Bt^2 + Ct^3}\right) \\
&= \frac{\left[\frac{d}{dt}(At)\right](Bt^2 + Ct^3) - \left[\frac{d}{dt}(Bt^2 + Ct^3)\right](At)}{(Bt^2 + Ct^3)^2} \\
&= \frac{(A)(Bt^2 + Ct^3) - (2Bt + 3Ct^2)(At)}{(Bt^2 + Ct^3)^2} \\
&= \frac{-ABt^2 - 2ACt^3}{(Bt^2 + Ct^3)^2} \\
&= -\frac{At^2(B + 2Ct)}{t^4(B + Ct)^2} \\
&= -\frac{A(B + 2Ct)}{t^2(B + Ct)^2}
\end{aligned}$$